

University of Washington Math Hour Olympiad, 2018

Grades 6-7

6. Seventeen teams participated in a soccer tournament where a win is worth 1 point, a tie is worth 0 points, and a loss is worth -1 point. Each team played each other team exactly once. At least $3/4$ of all games ended in a tie. Show that there must be two teams with the same number of points at the end of the tournament.



7. The city of Old Haven is known for having a large number of secret societies. Any person may be a member of multiple societies. A secret society is called *influential* if its membership includes at least half the population of Old Haven. Today, there are 2018 influential secret societies. Show that it is possible to form a council of at most 11 people such that each influential secret society has at least one member on the council.